

Testing Strong-field Gravity with Quasi-Periodic Oscillations

Simon DeDeo

Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey, 08544

Dimitrios Psaltis

Physics and Astronomy Departments, University of Arizona, 1118 E. 4th St., Tucson, AZ 85721

The frequencies of quasi-periodic oscillations around neutron stars are believed to be related to characteristic frequencies in the gravitational fields of the compact objects. In different variability models, these include the Keplerian, epicyclic, and Lense-Thirring frequencies, which depend mostly on the properties of the stellar spacetimes. We argue that quasi-periodic oscillations in the X-ray flux of neutron stars can be used to map the external spacetimes of the compact objects and, therefore, lead to direct tests of general relativity in the strong-field regime. In particular, we show that particular extensions of General Relativity, in which the gravitational force felt by matter is mediated by both a rank-two tensor and a scalar field, can be constrained by current observations.

I. INTRODUCTION

Neutron star properties, such as radius and mass, help probe the nature of strong-field gravity. In a previous paper [1], we showed how the measurement of the redshift of atomic lines emitted from the surface of a neutron star can be a powerful tool for investigating the properties of gravity in the strong-field regime. In particular, we found that our current lack of knowledge about the behavior of gravity in the strong-field regime far outweigh our current lack of knowledge about the neutron star equation of state.

In this paper, we expand that investigation to show how the properties of the spacetime around a neutron star are at least as sensitive to the choice of gravity theory in the range currently allowed by weak-field observations as they are to the exact nature of the neutron star equation of state. As in [1], we choose a particular single-parameter scalar-tensor theory but this time we extend our consideration to the spacetime outside the neutron star surface.

In particular, we examine the way in which deviations from general relativity may affect the frequencies of quasi-periodic oscillations (QPOs) found in many X-ray binaries. QPOs are of contested origin; we refer the reader to Refs. [2, 3]. All models, however, agree that the processes responsible for QPO production take place in a region of order the Schwarzschild radius around the compact object. This makes QPOs excellent probes of strong-field gravity, especially if, as many believe, their frequencies are directly related to the characteristic frequencies of test particles near the innermost stable circular orbit (ISCO). The growing numbers of QPO observations promises an ideal testing ground for the predictions of general relativity.

We provide two new methods of constraining deviations from general relativity with QPO observations. The first method relies only on general arguments about the maximum coherent frequency of the lowest-order modes near the surface of neutron stars [4]. The second method, which relies upon a particular interpretation [5] of a

commonly observed QPO phenomenon – the correlation in peak position found between high frequency QPO pairs [6, 7] – approaches the binary pulsar tests in its ability to constrain the particular scalar-tensor effect considered. The generality of our first method demonstrates the importance of examining a wide range of high-energy X-ray phenomena of diverse nature and origin for clues about the properties of strong-field gravity. The power of our second method demonstrates how the development of more comprehensive physical accounts of these phenomena can lead to new advances in our ability to use the universe as a gravity laboratory, probing spacetime curvatures many orders of magnitude higher than those previously accessible to precision gravity tests.

Our paper is organized as follows. In section two, we describe the field equations and stellar structure equations for stars in a general scalar-tensor theory and we solve them through a combination of analytic and computational techniques. In section three, we discuss in detail the previously unremarked-upon properties of test particle orbits around such stars and, in section four, we apply these results to show explicitly how QPO properties can be used to constrain the nature of strong-field gravity.

II. EQUATIONS OF STELLAR STRUCTURE

Following [8] as an example of a gravity theory that deviates from GR in the strong-field regime, we consider a theory containing a scalar field coupled to gravity with the action

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g_*} (R_* - 2g_*^{\mu\nu} \phi_{,\mu} \phi_{,\nu}) + S_m[\Psi_m, A^2(\phi)g_{*\mu\nu}]. \quad (1)$$

Here Ψ_m refers collectively to all matter fields other than ϕ , G_* is a dimensional constant, and $A(\phi)$ is a function of ϕ . The $(0, 2)$ tensor $g_{*\mu\nu}$ is the Einstein frame metric; all matter (apart from ϕ), flows on geodesics of the physical

(or “Brans-Dickie”) frame metric, $A^2(\phi)g_{*\mu\nu}$, which we will refer to as $g_{\mu\nu}$. The field equations for the scalar field ϕ are particularly simple in the Einstein frame where ϕ appears to be non-minimally coupled to gravity; because of the unphysical nature of the metric $g_{*\mu\nu}$, however, confusion may arise; the reader is referred to [9] for a deeper discussion of these issues. Unless otherwise specified, all quantities reported in this paper are measured in the physical frame.

We summarize the field equations, following [10]:

$$R_{*\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi + 8\pi G_* \left(T_{*\mu\nu} - \frac{1}{2}T_*g_{*\mu\nu} \right),$$

$$\nabla_{*\nu}\nabla_{*\mu}\phi = -4\pi G_*\alpha(\phi)T_*.$$

The equation of motion for matter is:

$$\nabla_{*\nu}T_{*\mu}^\nu = \alpha(\phi)T_*\nabla_{*\mu}\phi,$$

where starred subscripts refer to operations using the Einstein frame metric, and $\alpha(\phi) = d\ln A(\phi)/d\phi$. The stress energy tensor in the Einstein frame, $T_{*}^{\mu\nu}$, is equal to $A^6(\phi)T^{\mu\nu}$, where $T^{\mu\nu}$ is measured in the physical frame.

We see that the scalar field, ϕ , is sourced by the trace of the stress-energy tensor, proportional to the logarithmic derivative of the warp factor, $A(\phi)$. Thus, though the cosmological value of ϕ may be zero, a strong enough concentration of matter can support a non-zero value of ϕ . Given a sufficiently dense concentration of matter, and a sufficiently steep $A(\phi)$, a region of space with non-zero ϕ may become energetically favorable.

Solar system experiments constrain the function A to be very flat at the cosmological value of ϕ , denoted by ϕ_0 . If, however, the ϕ field inside a compact object fluctuates just far enough away from cosmological to discover a steep part of A , perhaps during a violent formation event, the system will be able to reach the more energetically favorable configuration, with large non-zero values for the scalar field ϕ near the center of the object. This is the phenomenon of “spontaneous scalarization.”

This phenomenon occurs *in general* for functions $A(\phi)$ when the derivative $d\ln A/d\phi|_{\phi_0}$ is sufficiently negative [10], although the precise configuration of the neutron star and its external spacetime may vary with the particular choice of $A(\phi)$. We consider only the first term in the Taylor expansion of $\ln A(\phi)$ that leads to negative curvature,

$$A(\phi) = e^{\frac{1}{2}\beta\phi^2}, \quad (2)$$

where β is a real number. This is the same form as used in [11], where it is referred to as the “quadratic model” because of its relationship to the Taylor expansion of $\ln A(\phi)$. Other investigations of scalar-tensor theory have chosen different forms for $A(\phi)$; see [12] for a comparative list of choices of $A(\phi)$, as well as for how to

rewrite the action in Eq. 1 explicitly for particular choices of $A(\phi)$ used in the literature, in both the Einstein and physical frame.

Again following [10], we can write the relativistic equations of stellar structure for a neutron star. We write our metric as:

$$ds_*^2 = -e^{\nu(r)}dt^2 + \left[1 - \frac{2\mu(r)}{r}\right]^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

and describe the matter fields as a perfect fluid in the physical frame:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}.$$

The differential equations to be integrated are then:

$$\mu' = 4\pi G_* r^2 A^4 \rho + \frac{1}{2}r(r - 2\mu)\psi^2, \quad (4)$$

$$\nu' = 8\pi g_* \frac{r^2 A^4 p}{r - 2\mu} + r\psi^2 + \frac{2\mu}{r(r - 2\mu)}, \quad (5)$$

$$\phi' = \psi, \quad (6)$$

$$\psi' = 4\pi G_* \frac{r A^4}{r - 2\mu} [\alpha(\rho - 3p) + r(\rho - p)\psi + \frac{\mu}{r(r - 2\mu)}\psi], \quad (7)$$

$$N' = 4\pi n A^3 r^2 (1 - \frac{2\mu}{r})^{-1/2}, \quad (8)$$

$$p' = -(\rho + p) \left[4\pi G_* \frac{r^2 A^4 p}{r - 2\mu} + \frac{1}{2}r\psi^2 + \frac{\mu}{r(r - 2\mu)} + \alpha\psi \right]. \quad (9)$$

Here N is the baryon number, n the number density, ρ the density, and p the pressure. We supplement these equations with a (zero temperature) equation of state, $p = p(\rho)$ and $n = n(\rho)$. Primes denote derivatives with respect to coordinate radius r .

Having chosen an equation of state, we then integrate Eqs. 4–9 from the center of the star, where we specify

$$\begin{aligned} \mu(0) &= \nu(0) = 0, \\ \phi(0) &= \phi_c, \\ p(0) &= p_c, \\ N(0) &= n_c, \\ \rho(0) &= \rho_c, \end{aligned}$$

to the surface, where $p = 0$. We then must integrate the equations for ν , ϕ , ψ , and μ from the surface of the star to infinity, to determine the form of the metric exterior to the star.

Throughout this paper we will be interested only in solutions where ϕ_0 , the cosmological value of ϕ , is zero

[and $A(\phi_0) = 1$, which can be achieved by renormalizing A]. We create our set of solutions to Eqs. (4)–(9) with a simple shooting method [13]. We note that μ *does not* have compact support; outside of the star the scalar field ϕ is non-zero (though relaxing as $1/r$ to zero) and contributing to the physical Arnowitt-Deser-Misner (ADM) energy felt by an observer far away. In our terminology, the physical ADM energy is that that would be inferred, e.g., from test particle orbits near infinity, and is identical to the M_{ADM} in [9]. When considering solutions to the metric, we renormalize the lapse function, $\exp[\nu(r)]$ so that the physical metric, Eq. (3), goes to $\eta_{\mu\nu}$, the Minkowski value, at infinity.

In an earlier paper [1], we considered three commonly used equations of state, which cover a broad subset of the wide range discussed in Cook et al. [14]. In order of increasing stiffness, these were EOS A [15], EOS UU [16], and EOS L [17]. Neutron star models computed with these three equations of state bracket the uncertainty introduced by our inability to calculate from first principles the properties of ultra-dense matter, when condensates or unconfined u-d-s quark matter is not taken into account.

For this paper, we will use the “median” EOS UU to illustrate the qualitative behavior of the variables we wish to examine; the results calculated for UU differ only slightly from those calculated with the two bracketing equations of state, A and L, in accord with the results presented in [1].

For simplicity, we neglect the effect of rotation on the frequencies of orbits in the external spacetime. In General Relativity, the effect of rotation on the orbital frequencies is less than 20% for the inferred spin frequencies of the neutron stars that show QPOs. We expect rotation to affect the conclusions in this paper at the same level.

III. PROPERTIES OF SCALAR STARS

Our gravity theory is described by the action in Eq. (1) and the definition of the function $A(\phi)$ in Eq. (2). The function $A(\phi)$ has one free, real parameter, β , which characterizes the strength of the scalar field coupling to gravity.

Spontaneous scalarization occurs only when $\beta \leq -4.85$. In Fig. 1, we plot the binding energy, $(Nm_b - M_{\text{ADM}})$, where m_b is the baryon rest mass), as a function of central density ρ_c , using equation of state UU, for a range of values for the parameter β . We draw the reader’s attention to the very high binding energies that can be achieved when β is sufficiently large. This large energy is due mostly to energy released when ϕ drops away from its cosmological value; when a scalarized star is produced, this energy goes into producing a monopole “gravitational wave” in ϕ .

After the “turnover point”, *i.e.*, the central density for which the mass for a particular choice of β is at its maximum, the stars are unstable to radial perturbation and

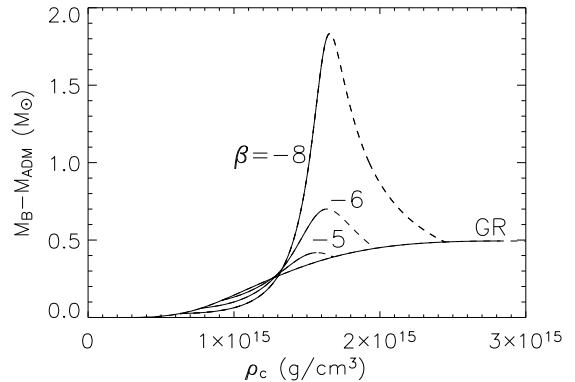


FIG. 1: Binding energies of our neutron star configurations, as a function of central density, for different values of the parameter β , for equation of state UU. Dashed lines indicate where a star is unstable to radial perturbations.

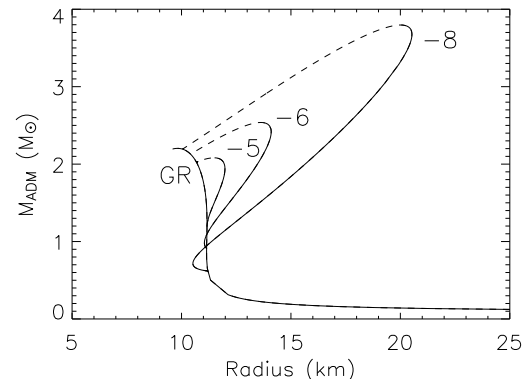


FIG. 2: The mass-radius relation for our neutron star configurations, for different values of the parameter β , for equation of state UU. Dashed lines indicate where a star is unstable to radial perturbations. A fiducial value comparable to the most accurately measured neutron-star masses is drawn as a horizontal heavy dashed line.

will collapse; these stars are marked with a dashed line in Fig. 1. Following the stability analysis of [18], we know also that when the binding energy for a scalarized star is greater than that for its unsclalized, General Relativistic counterpart with the same central density, *only* the scalarized configuration is stable. Scalarization is mandatory when the scalarized star has greater binding energy.

Fig. 2 shows the mass-radius relationship for the same range of the parameter β and the same equation of state, UU, as above. Immediately apparent is the result [9, 10] that scalarized stars can achieve large masses and large radii.

IV. ORBITS IN THE VICINITY OF SCALAR STARS

Here we consider the properties of a test particle in a circular orbit outside the star. We first consider the orbital frequency of the particle as seen at infinity. Paralleling the analysis for circular orbits in the Schwarzschild metric [19], we find:

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{(1 - 2\mu/r)}{A^2} \left[\frac{\tilde{E}^2}{A(\phi)^2 e^\nu} - \left(1 - \frac{\tilde{l}^2}{A(\phi)^2 r^2}\right) \right],$$

where \tilde{E} and \tilde{l} are the energy and angular momentum per unit mass of the test particle, respectively, τ is the proper time of the particle, and r is the orbital radius. We note that, in contrast to the General Relativistic case, μ is a function of radius (as is, necessarily, A , which is a function of ϕ). For the orbit to be circular, we must have

$$\begin{aligned} \left(\frac{dr}{d\tau}\right)^2 &= 0, \\ \frac{d}{dr} \left(\frac{dr}{d\tau}\right)^2 &= 0. \end{aligned}$$

Using these conditions to eliminate \tilde{E} and \tilde{l} , and using the lapse function $dt/d\tau$, we find the orbital frequency of a test particle, as measured at infinity, to be

$$\begin{aligned} \omega_o &= \frac{d\phi}{dt} = e^{\nu/2} \sqrt{\frac{2\beta\phi\phi' + \nu'}{2\beta\phi\phi' r^2 + 2r}} \\ &= \sqrt{\frac{\mu}{r^3} \left[\frac{e^\nu(1 - 2\mu/r)^{-1} + \psi(\beta\phi + r\psi/2)r^2 e^\nu/\mu}{1 + \psi r\beta\phi} \right]}, \end{aligned}$$

where, as before, primes denote derivative with respect to r . The final form of this equation demonstrates how the orbital frequency approaches the general relativistic value as the scalar coupling, β , is taken to zero. In GR, $\psi = \phi = 0$ and $e^\nu(1 - 2\mu/r)^{-1} = 1$, so that we obtain the Kepler frequency $\omega^2 = \mu/r^3$.

Figure 3 shows the ratio of orbital frequency to the “Keplerian” value as a function of the logarithm of the distance from the center of mass, for a star with $M_{\text{ADM}} = 1.5M_\odot$, for equation of state UU and for a range of values of the parameter β . We define the Keplerian frequency at radius r to be the orbital frequency that would be produced at radius r were the scalarized star replaced by a General Relativistic star with M_{ADM} equal to $\mu(r)$. We remind the reader that $\mu(r)$ is increasing even outside the star, since ϕ is non-zero near the surface.

Comparison of figures 3 and 2 reveals an interesting and counter-intuitive effect of spontaneous scalarization. Scalarized stars have radii much greater than their

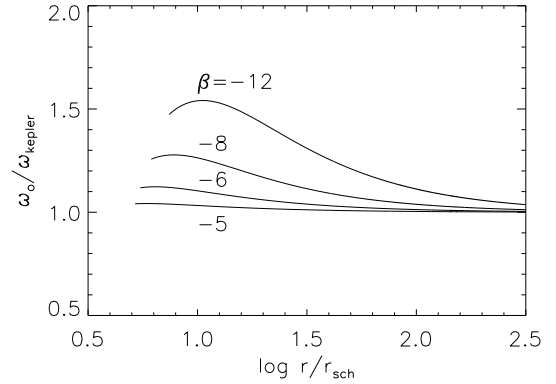


FIG. 3: The ratio of orbital to “Keplerian” frequencies for a star with $M_{\text{ADM}} = 1.5M_\odot$ for a range of values of β , using equation of state UU. Not all the orbits for which frequencies are plotted here are stable; see Fig. 4.

General Relativistic counterparts with the same M_{ADM} . One would thus presume that their surface gravity, and thus their orbital frequencies near the surface, would be lower than the General Relativistic case. In fact, the case is quite the opposite; near the surface of a scalarized star the orbital frequency can be up to 50% higher than they would be at the same distance for a General Relativistic star with the same enclosed mass. At distances greater than ≈ 300 Schwarzschild radii, however, the orbits converge rapidly to their General Relativistic values. This is a dramatic illustration of how large strong-field effects can effectively hide themselves in a small region.

To investigate the stability of these orbits to radial perturbations we calculate the epicyclic frequency κ . We find

$$\kappa^2 = \frac{1}{2} \left(\frac{d\tau}{dt}\right) \frac{d^2}{dr^2} \left(\frac{dr}{d\tau}\right)^2.$$

The orbital stability criterion is simply that $\kappa^2 > 0$. The form of the right hand side of this equation is complicated, but it depends only on the variables in Eqs. (4)–(9) and their derivatives.

Figure 4 shows the behavior of the epicyclic frequency as a function of the logarithm of radius, again with $M_{\text{ADM}} = 1.5M_\odot$, equation of state UU, and a range of values for the parameter β . A negative epicyclic frequency is used here to indicate an imaginary κ (i.e., $\kappa^2 < 0$, instability.) We see again how the properties of the scalarized stars are altered by factors of order unity compared to the General Relativistic case. The point of instability is changed such that, as the parameter β is made more negative, the ISCO radius is pushed closer to the surface of the star compared to the GR ISCO radius. As expected, for radii much larger than the ISCO or stellar radius, the ratio of epicyclic to orbital frequency converges to the General Relativistic value.

As can be seen in Fig. 4, for values of the parameter

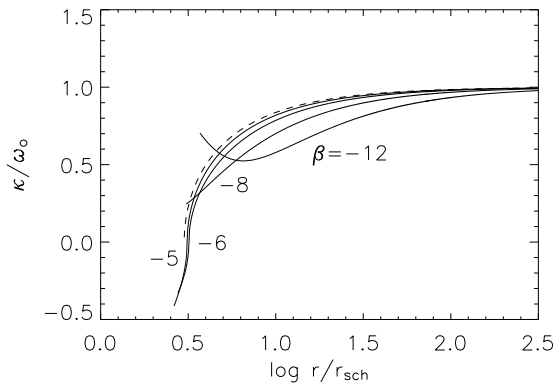


FIG. 4: The ratio of epicyclic to orbital frequencies for a range of values of β and $M_{\text{ADM}} = 1.5M_{\odot}$, using equation of state UU. A negative epicyclic frequency is used here to indicate instability (i.e., $\kappa^2 < 0$). The limiting GR case is plotted as a dashed line.

$\beta \lesssim -8$ and neutron star masses $M_{\text{ADM}} \sim 1.5M_{\odot}$, all orbits down to the stellar surface are stable.

V. CONSTRAINTS ON β FROM QUASI-PERIODIC OSCILLATIONS

The predicted differences in orbital and epicyclic frequencies between the General Relativistic and scalar-tensor theories discussed in the previous section suggest a number of possible tests that can be conducted using the observed properties of quasi-periodic oscillations in X-ray binaries. In this section, we will discuss two such methods. The first, relying only on the difference between the General Relativistic and scalar-tensor orbital frequencies, will turn out to be an as-yet inconclusive – though rather general – test. The second, which relies on a proposed theoretical picture of QPOs, generates tighter constraints.

A. Limiting β by orbital frequency alone

In many QPO models, a strong peak in the Fourier spectrum of the observed luminosity time-series is produced near the orbital frequency of the inner edge of the Keplerian disk [3]. In both General Relativistic and scalar-tensor theories, the orbital frequency increases monotonically with decreasing distance from the center of the object.

Thus, assuming that the QPO frequencies come from some radius close to but never less than that of the inner edge of the Keplerian flow where the epicyclic frequency becomes imaginary, this suggests a simple null test of scalar tensor theories. To wit: is the highest possible orbital frequency one can produce around a scalar-tensor star – for a particular value of the parameter β – lower

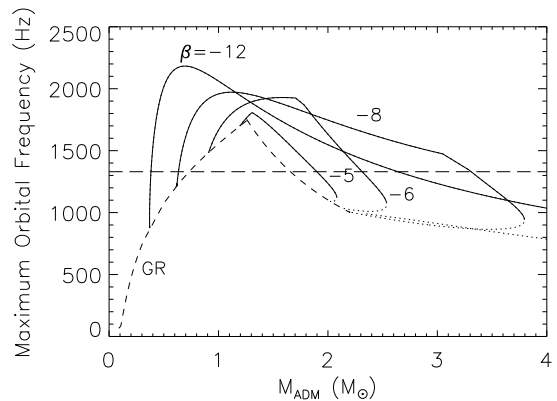


FIG. 5: The highest orbital frequency produced in scalar-tensor theories, as a function of mass, for equation of state UU. The long dashed line shows the current 1330 Hz lower limit on the maximum QPO frequency, the short dashed line represents the General Relativistic case, and the dotted lines indicate unstable branches.

than any observed QPO frequency? We may refine this question further by specifying a reasonable (ADM) mass range we wish the scalar-tensor stars to have.

The current lower limit on the maximum QPO frequency is 1330 Hz [20]. We draw this lower limit on Fig. 5, where we show the highest orbital frequency for a variety of scalar-tensor theories as a function of mass, for equation of state UU. The nonintuitive property of scalar-tensor stars can immediately be seen: making the parameter β more negative increases the star radius (see Fig. 2) but also increases the orbital frequency at every radius beyond the stellar surface (see Fig. 3); this means that a wider range of masses are able to produce the high maximum frequency in scalar-tensor theory. There appears to be no simple way of demonstrating the necessity of this result; all that can be said is that the metric around a scalar-tensor star behaves very differently from its Schwarzschild cousin.

For completeness, we present this result in a complementary manner in Fig. 6. Here we show, as a function of the value of β , the mass range for a variety of equations of state that can reproduce the QPO lower limit of 1330 Hz. Thus this test, though ideal because of its model independent nature, cannot rule out any part of the parameter space.

B. Limiting β by joint analysis of orbital and epicyclic frequencies

The high-frequency QPOs discussed in this paper often come in pairs [2] with the two peaks maintaining slowly changing separations. The relativistic precession model [5] predicts a simple relationship between, on the one hand, the positions and separations of these QPO peaks and, on the other hand, the orbital and epicyclic

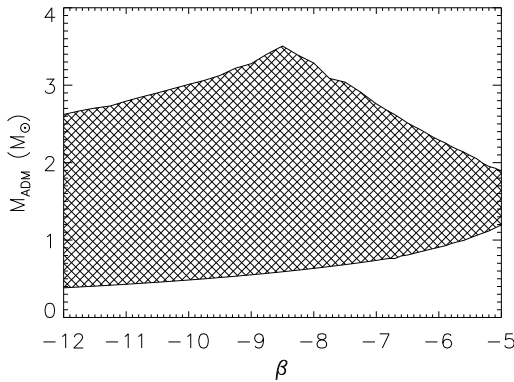


FIG. 6: Allowed range (indicated by crosshatching) for M_{ADM} , as a function of β , for EOS UU, for the current best lower limit on the maximum QPO of 1330 Hz.

frequencies at the inner edge of the accretion disk of the neutron star.

In this model, as the inner radius of the disk changes over time, the QPO frequencies change to reflect the change in orbital and epicyclic frequency. This relationship allows one to use measurements from a single object to sample, not just a single radius in the neutron star external spacetime, but a whole region of the orbital plane. This results in much stronger constraints on the nature of strong-field gravity while at the same time reducing the sensitivity to the choice of equation of state for ultra-dense matter. As we will see, this allows us, along with the assumption that the stars are slowly-rotating, to rule out the scalar-tensor theory of gravity contained in Eqs. (1) and (2) for a range of the parameter β .

To derive constraints on the scalar-tensor theory we first solve the equations of motion for particles in the external spacetime, for each value of the parameter β and M_{ADM} , to find the Keplerian orbital frequency, ω_o , and the epicyclic frequency, κ , as a function of radius. The lower bound for the radius is either that of the innermost stable orbit, or that at the surface of the neutron star itself, whichever is larger. We then find the relationship between ω_o and $\omega_o - \kappa$ themselves, which according to the model correspond to the frequencies of the two QPOs.

We then fit this relationship to the observed QPO frequency pairs, and derive a reduced chi-squared value for each point in (M_{ADM}, β) space. In Fig. 7, we plot the two-sigma confidence contour around the minimum χ^2 value obtained for the GR case. As can be seen, the onset of scalarization alters the range of allowed masses, and for values of $\beta \lesssim -10.25$, scalarization produces neutron stars with spacetimes sufficiently different from the GR case as to be ruled out altogether.

We emphasize that this test has assumed that the rotation of the neutron star itself has a small (less than order unity) effect on the external spacetime. While we have not constructed and solved the equations for the scalar-

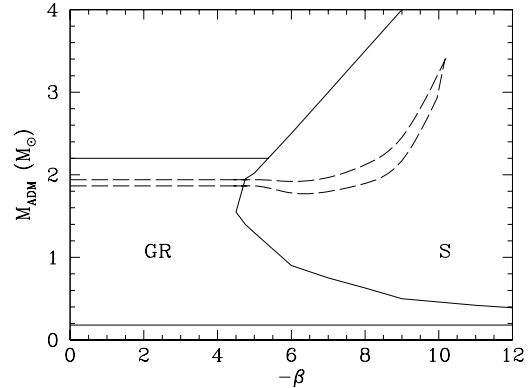


FIG. 7: Two-sigma confidence contours (dashed line), as a function of M_{ADM} , for a fit to the data of Ref. [21], for a range of values of β and EOS UU. The regions bounded by solid lines show the region of standard GR behaviour (“GR” region) and the parameter space in which stars undergo spontaneous scalarization (“S” region.)

tensor theory in the case of a rotating matter source, we believe that the corrections to the orbital-epicyclic frequency relationship which governs this test will be on the order of 20%.

We expect that combining observations from different systems, and requiring that β be held fixed across all observations, will provide additional confirmation of this result. Because of the uncertainty introduced by the consideration of nonzero stellar angular momentum, we cannot put a precise lower bound on the value of the parameter β , but given the magnitude of corrections found for rotation in the GR case, the lower bound is presumably not far from $\beta \sim -10$.

The analysis of this section has relied on a particular model of QPO production. Other QPO models, since they also invoke processes at the high spacetime curvatures near the surface of the star, will lead to similar kinds of constraints, of varying severity, for scalar-tensor models.

VI. DISCUSSION

We have investigated two different methods for constraining the nature of strong-field gravity using observations of QPOs. As an example of a theory of gravity that deviates from GR, we have used a particular scalar-tensor theory, parametrized by a single real number, β . The class of theories we have investigated has gained a great deal of attention because of its invisibility to traditional, weak-field tests.

We have investigated the nature of test particle orbits in the external spacetime of the neutron star, and, while uncovering some counterintuitive behavior, have confirmed for test particle orbits what has already been

known for other observables of compact objects in this scalar-tensor theory: while large (order unity or greater) deviations from General Relativity are observed within the first few hundred Schwarzschild radii of the neutron star, the effects of introducing a scalar field rapidly disappear at the larger distances (and far lower curvatures) probed by ordinary weak-field tests and require observations of much greater precision to detect.

The first method we have investigated, while not relying on a particular theory of QPO production, does not produce strong constraints on the value of the parameter β . The second method, which relies upon the relativistic precession model of QPO production, is more powerful in its ability to separate the predictions of General Relativity from those of scalar-tensor theory. The power of this second method derives in part from the fact that we are able to sample a range of radii around the neutron star, and not just a single radius.

In a previous paper [1], we investigated the use of gravitationally redshifted atomic lines in testing strong-field gravity, and showed that the nature of gravity in the strong-field is less well known than the equation of state for ultra-dense matter. Further constraining this equa-

tion of state has often been taken to be the goal of theoretical studies of neutron star properties.

We find, however, that a second, promising avenue is opening up for investigations that attempt to constrain the nature of gravity itself. In addition to the tests presented here and in previous work, additional ways of testing strong-field gravity can be performed that take advantage of other aspects of the rapid observational and theoretical progress that has been made in recent years in the study of compact objects in the X-ray and soft-gamma ray bands. Another such test based on the Eddington-limited luminosities of radius-expansion bursts will be reported [22].

While solar system and binary pulsar tests observe with a much higher degree of precision than can be expected from the investigation of high-energy processes near the surface of compact objects, the large spacetime curvatures found in the latter systems can, in some circumstances, magnify the expected deviations and produce constraints on the nature of GR of similar severity. In such cases, weak field tests and strong-field tests described in this paper and others may be considered complementary.

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- [1] S. DeDeo and D. Psaltis, Phys. Rev. Lett. **90**, 141101 (2003).
 - [2] M. van der Klis, Ann. Rev. Astron. Astrop. **38**, 717 (2000).
 - [3] D. Psaltis, Advances in Space Research **28**, 481 (2001).
 - [4] M. C. Miller, F. K. Lamb, and D. Psaltis, Astrophys. J. **508**, 791 (1998).
 - [5] L. Stella, M. Vietri, and S. M. Morsink, Astrophys. J. **524**, L63 (1999).
 - [6] D. Psaltis, T. Belloni, and M. van der Klis, Astrophys. J. **520**, 262 (1999).
 - [7] T. Belloni, D. Psaltis, and M. van der Klis, Astrophys. J. **572**, 392 (2002).
 - [8] T. Damour and G. Esposito-Farèse, Class. Quantum Grav. **9**, 2093 (1992).
 - [9] M. Salgado, D. Sudarsky, and U. Nucamendi, Phys. Rev. D **58**, 124003 (1998).
 - [10] T. Damour and G. Esposito-Farèse, Phys. Rev. Lett. **70**, 2220 (1993).
 - [11] T. Damour and G. Esposito-Farèse, Phys. Rev. D **54**, 1474 (1996).
 - [12] A. W. Whinnett, Phys. Rev. D **61**, 124014 (2000).
 - [13] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C* (Cambridge University Press, 1993).
 - [14] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky, Astrophys. J. **424**, 823 (1994).
 - [15] V. R. Pandharipande, Nucl. Phys. A **174**, 641 (1971).
 - [16] R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).
 - [17] V. R. Pandharipande and R. A. Smith, Phys. Lett. B **59**, 15 (1975).
 - [18] T. Harada, Phys. Rev. D **57**, 4802 (1998).
 - [19] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (San Francisco: W.H. Freeman and Co., 1973, 1973).
 - [20] S. van Straaten, E. C. Ford, M. van der Klis, M. Méndez, and P. Kaaret, Astrophys. J. **540**, 1049 (2000).
 - [21] M. Méndez and M. van der Klis, MNRAS **318**, 938 (2000).
 - [22] S. DeDeo and D. Psaltis (2004), in preparation.